

HOLOGRAPHIC CALCULATION OF THE π MESON-OCTET BARYON COUPLING CONSTANT

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Abstract: The Anti-de Sitter/Conformal Field Theory (AdS/CFT) correspondence between theories in AdS space and conformal field theories in physical space-time provides as analytic, the semi-classical model for strongly-coupled QCD. In this work, we investigate properties of the octet baryon in the holographic model, calculate the coupling constant between π meson-octet baryon in the framework holographic model by considering the infrared-modified 5D conformal mass and Yukawa coupling of the bulk baryon field. Predictions of this model for the mass spectrum of the baryon octet are appropriate with the experimental value. It means the hard-wall AdS/QCD model is favorable for determining coupling constants between elementary particles.

Keywords: AdS/CFT correspondence, hard wall model, meson, baryon, coupling constant.

1. Introduction

To calculate the coupling constants, form factors of the elementary particles in the framework AdS/QCD models opens a new opportunity in hadronic physics. Because, these models can study essential properties of QCD such as confinement and chiral symmetry breaking and have demonstrated in many cases success in the determination of static hadronic properties, such as resonance masses, decay constants.

The AdS (Anti-de Sitter) space-time is the space-time of constant negative curvature and has the SO(2,4) invariance [6]. This symmetry is the same as the (3+1)-dimensional conformal invariance of the N=4SYM. The AdS/CFT (Anti-de-Sitter/Conformal Field Theory) correspondence states that there is an exact equivalence between string theory on asymptotically AdS space-times and a quantum field theory that “resides” on the conformal boundary of the AdS space-time.

Much research has been focused to build a holographic model of baryons based on the AdS/QCD models. In Ref[2], the authors calculate mass spectra of the baryon octet and their low-lying excited states. Experimental value compatible model predictions are compatible with the experimental value, in particular, the ground states for the mass spectrum of the baryon octet.

For simplicity, the space-time geometry in bottom-up models is often chosen to be a slice of AdS₅ with metric given by Eq. (1) between $z = \epsilon$ and $z = z_m$ where, $\epsilon/z_m \ll 1$. Such models are called hard-wall models. Because of the sharp boundary $z = z_m$ [4] and this letter aims to study mass spectrum of the baryon octet and coupling constants between π –meson-octet baryon in the framework of the hard-wall AdS/QCD model.

2. The model

The model uses an AdS space with a hard-wall cutoff in the IR as the holographic 5D background. The metric is:

$$ds^2 = g_{MN}dx^M dx^N = \frac{1}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - z^2). \quad (1)$$

The 4-dimensional metric $\eta_{\mu\nu}$ has Minkowski signature: $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$. The model is defined in the range: $\epsilon \leq z \leq z_m$.

For the hard-wall model S_{int} is defined as following:

$$S_{int} = \int_0^{z_m} d^5x \sqrt{G} L_{int}. \quad (2)$$

Formally define the quantum gravity partition function with field values at the AdS boundary through the path integral [8]:

$$Z_{AdS-QGr}[\lambda_i(x, y_0)] = \int D\lambda_i e^{S_{QGr}}, \quad (3)$$

Where, the path integral is only on the field configurations taking value $\lambda_i(x, y_0)$ at $z = y_0$. The formal, but precise expression of AdS/CFT duality is then will be written as:

$$Z_{AdS-QGr}[\lambda_i(x, y_0)] = Z_{CFT}[\lambda_i(x, y_0)]. \quad (4)$$

Both corrections are small one may safely replace S_{QGr} in by S_{SUGRA} (on AdS). Moreover, since quantum effects are also small the RHS of (3) may be well approximated, in the WKB/saddle point approximation, by the value of the exponential of classical gravity action with field values satisfying classical gravity field equations with boundary conditions $\lambda_i = \lambda_i(x, y_0)$. Explicitly,

$$Z_{CFT}[\lambda_i(x, y_0)] \cong e^{iS_{on-shell}^{gravity}}. \quad (5)$$

The above is statement of gauge/gravity correspondence.

Expression of the baryon current is vital to calculate coupling constant. The current is expressed below in the boundary QCD theory [3]:

$$\langle J \rangle = -i \frac{\delta Z_{AdS}}{\delta \hat{P}^0} \Big|_{\hat{P}^0=0}. \quad (6)$$

Couplings of elementary particles are found using explicit expression in AdS/QCD models:

$$J(p', p) = g_{\pi\Lambda} \bar{u}(p') \gamma^5 \frac{\tau^a}{2} u(p). \quad (7)$$

3. Octet baryon in holographic model

Chiral symmetry breaking can be formulated in the action below:

$$S_{b2} = - \int d^5x \sqrt{g} [c_1 \text{Tr}(\bar{B}_1 \{\chi_+, B_2\}) + c_2 \text{Tr}(\bar{B}_1 [\chi_+, B_2]) + (c_2 - c_1) \text{Tr}(\bar{B}_1 B_2) \text{Tr} \chi_+] \quad (8)$$

It is related to the three-flavor generalization of the bulk scalar field X by:

$$\xi_+ = \frac{1}{2}(\xi^+ X \xi^+ + \xi X^+ \xi). \quad (9)$$

X is the bulk scalar field and this is defined as follow:

$$X = \xi(X_0 + T_0 S_0 + T_a S_a)\xi, \quad (10)$$

Where, S_0, S_a represent the scalar and octet., X_0 is the vacuum expectation value of the scalar field:

$$X_0 = \frac{1}{2} \begin{pmatrix} v_u & 0 & 0 \\ 0 & v_u & 0 \\ 0 & 0 & v_s \end{pmatrix}. \quad (11)$$

v_u, v_s can be written as below

$$\begin{aligned} v_u &= m_u z + \sigma_u z^3, \\ v_s &= m_s z + \sigma_s z^3, \end{aligned}$$

Where, m_u, m_s are the masses of the u and d quarks, σ_u, σ_s are chiral condensate of u and d quarks.

If (9) (10) (11) are taken into account in (8) expression, $v(z)$ expression will be as follow for different particle in the octet baryon [13]:

$$v(z) = (3c_2 - c_1)v_u(z) \quad \text{for } (\mathbf{p}, \mathbf{n}) \quad (12)$$

$$v(z) = \left(2c_2 - \frac{4c_1}{3}\right)v_u(z) + \left(\frac{c_1}{3} + c_2\right)v_s(z) \quad \text{for } \Lambda \quad (12')$$

$$v(z) = 2c_2 v_u(z) + (c_2 - c_1)v_s(z) \quad \text{for } \Sigma_S \quad (12'')$$

$$v(z) = (c_2 - c_1)v_u(z) + 2c_2 v_s(z) \quad \text{for } \Xi_S. \quad (12''')$$

4. π -Meson octet baryon coupling constant

The excited states of baryons form singlets, octets or decuplets nearly degenerate in parity. The octet baryons are conventionally embedded in the matrix

$$B_j^i = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}.$$

As in Ref [1], the bulk baryon fields B_j can be written in the chiral form

$$B_j = B_{jL} + B_{jR} \quad (13)$$

The $B_{jL,R}$ chiral components are written in the form:

$$B_{jL,R}(x, z) = \frac{1}{(2\pi)^4} \int d^4p f_{jL,R}(z) \psi_{L,R}(p) e^{-ipx}, \quad (14)$$

where, $f_{jL,R}(z)$ -are the Kaluza-Klein profiles of the baryon fields, $\psi_{L,R}(p)$ are the 4D spinors.

In QCD we know that the bulk of the baryon mass comes from chiral-symmetry breaking. Since the condensate $\langle \bar{q}q \rangle$ is the order parameter of chiral-symmetry breaking, the baryons should get mass through coupling to the condensate. In AdS/QCD it was shown that this can be easily achieved by introducing Yukawa couplings between bulk spinors and bulk scalar:

$$\mathcal{L}_{Yukawa} = -g_Y[\bar{B}_1 X B_2 + \bar{B}_2 X^+ B_1], \quad (15)$$

Where, g_Y -Yukawa coupling, B_1 and B_2 baryons are 5-dimensional spinors and $B_{iL,R}$ are decomposed by the Kaluza-Klein and Fourier decomposition. $B_{jL,R}$ is defined as (14) formula.

If to take into account the formulas (14) and (15) in the action (2), finally, it will get a form:

$$S = \frac{1}{2} \int_0^{z_m} dz \sqrt{G} \int d^4 p \int d^4 p' e^{i(p-p')x} v(z) P_\pi(p, z) (f'_{2L} f_{1R} - f'_{2R} f_{1L} - f'_{1L} f_{2R} + f'_{1R} f_{2L}) \bar{u}(p') \gamma^5 \frac{\tau^a}{2} u(p). \quad (16)$$

From this action the coupling constant between π meson- octet baryon can be written as following:

$$g_{\pi\Lambda} = \int_0^{z_m} \frac{dz}{z^5} v(z) P_\pi(z) g_Y (f'_{2L} f_{1R} - f'_{2R} f_{1L} - f'_{1L} f_{2R} + f'_{1R} f_{2L}) \quad (17)$$

5. Numerical analysis

In this section we present the numerical analysis of the π meson-octet baryon coupling constant. Numerical values of some parameters are taken as below[13]:

$$m_\pi = 0.14 \text{ (GeV)}, 3c_2 - c_1 = 14.8, m_u = 0, z_m = (205 \text{ MeV})^{-1}.$$

Furthermore, we take $\sigma_s = \sigma_u = (198 \text{ MeV})^3$ and the last parameter c_2 was fixed by the mass of Λ baryon to be $c_2 = 4.52$. We find formula of the $v(z)$ using above numerical values for octet baryon.

In the end, π meson-octet baryon coupling constant is computed these numerical value taking into account in ‘‘MATHEMATICA11’’. The experimental and numerical results for the octet-baryon spectrum are shown in Table1.

Table 1. Experimental data and model results for the mass spectrum of the baryon octet.

Baryon octet	P,n	Λ	Σ_s	Ξ_s
Exp.(MeV)	939	1115	1190	
Model(MeV)	939	1213	1004	

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ГОЛОГРАФИЧЕСКИЙ РАСЧЕТ КОНСТАНТЫ СВЯЗИ π МЕЗОНА С ОКТЕТНЫМИ БАРИОНАМИ

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Резюме: В данной работе мы исследуем свойства октетного бариона в голографической модели, вычисляем постоянную связи π мезонно-октетного бариона в базовой голографической модели рассматривая модифицированную инфракрасную массу и Юкавскую связь объемного барионного поля. Предсказания этой модели для масс-спектра барионного октета соответствуют экспериментальному значению. Это означает, что АдС/КПТ жесткой стенкой подходит для определения констант связи между элементарными частицами. Численные расчеты сделаны в программе «Mathematica».

Ключевые слова: АдС/КПТ соответствие, модель жесткой стены, мезон, барион.

π MEZON-OKTET BARIYON QARŞILIQLI TƏSİR SABİTİNİN HOLOQRAFİK HESABLANMASI

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Xülasə: Bu işdə biz holoqrafik modeldə oktet bariyonların xüsusiyyətlərini araşdırmışıq, π mezon-oktet bariyon qarşılıqlı təsir sabitini infraqırmızı modifikasiya edilmiş 5ölçülü kalibr kütləni və iç bariyon sahəsinin Yukava qarşılıqlı təsirini nəzərə alaraq holoqrafik model çərçivəsində hesablamışıq. Bu modelin oktet bariyonun kütlə spektrinin nəticələrilə təcrübi qiymətlərə yaxın olur. Bu o deməkdir ki, AdS/KXD-nin sərt divar modeli elementar zərrəciklər arasındakı qarşılıqlı təsir sabitlərinin hesablanması üçün əlverişli üsuldür. Nəzəri hesablamalar “MATHEMATICA11” proqramında aparılmışdır.

Açar sözlər: AdS/KSN uyğunluğu, sərt-divar modeli, mezon, bariyon.