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## THE POTENTIAL MODEL OF INTERACTION BETWEEN NUCLEON ASSOCIATIONS

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**Abstract:** The cluster model is applied to the study of associative knocking out nucleons in the  $A(p, pX)$   $A-X$  processes. The study is carried out with plane waves by Gaussian potential in the impulse approximation. It has been suggested that the scattering matrix does not depend on the transfer momentum. Partial amplitudes are invariant due to  $SU(4)$  conversion. The internal structure of the associations is neglected and effects of the Pauli principle between the nucleons in the associations are taken into account by introducing short-range repulsion between the associations. The spin-isospin flip of associations does not lead to recharging. The classification of states of association according to the orbital schemes of Jung automatically leads to orthogonality of wave functions with different partitions of associations.

**Keywords:** nucleon associations, scattering, impulse approximation, spin-orbital interaction, Gauss potential.

### 1. Introduction

The idea of nucleons clustering has a history back to the early thirties of the last century. By observing alpha decay from nucleus, people speculated that nuclei are made up of alpha particles. In his paper, Bethe [1] predicted that nuclei are made of alpha particles and gave also a geometrical arrangement of alpha particles inside nuclei. He predicted that, for each new addition of alpha particles, the number of increased bonds is three. At that time, this was too big challenge for the clustering models and eventually these models disappeared from the nuclear structure theories. Clustering is a recurrent feature in light nuclei, from beryllium to nickel. In light nuclei, the nucleons have been observed to cluster together forming sub-structures within the atomic nucleus, for states where the nucleons are just bound together. Association structures are typically observed as excited states close to the corresponding decay threshold; the origin of this phenomenon lies in the effective nuclear interaction, but the detailed mechanism of clustering in nuclei has not yet been fully understood. If we are taking into account the interaction between nucleons (as it is in reality) in the model approach, we can expect the formation of clusters. Calculations show that, the associations occur on the surface of the nucleus, where the density of nuclear matter less than in the downtown nucleus [2].

In this paper, we obtain an expression for the cross section of the ejection of nucleon associations based on Gaussian potential. We considered that, when the energy region lies below the threshold meson production, the impulse approximation is used.

### 2. The cross section of ejection of associations

We will neglect the internal structure of the associations and effects of Pauli principle between the nucleons in the clusters are taken into account by introducing short-range repulsion between the associations. The orthogonality condition of model and excluded state of model treat the clusters as elementary particles, but include effects of the Pauli principle in more microscopic way. Wheeler's resonating group method is a fully microscopic theory for calculating properties

of cluster systems. It makes simplifying assumptions about the internal structure of the clusters but takes the Pauli principle explicitly [3].

We can divide wave function for  $I+N$  nucleons into 2 associations as follows  $A-X$  and  $pX$  [4]. The wave function of the nucleons  $(1, \dots, m)$  in first cluster is  $\Psi_{A-X}(\mathbf{r}_1, \dots, \mathbf{r}_m)$  and the wave function of the nucleons  $(m+1, \dots, N+1)$  in second cluster is  $\Psi_{pX} = \Psi_{pX}(\mathbf{r}_{m+1}, \dots, \mathbf{r}_{N+1})$ . The total wave function is the antisymmetrized product

$$\Psi_A = \Psi_{A-X} \Psi_{pX} \chi(\mathbf{R}_1 - \mathbf{R}_2). \quad (1)$$

The vectors  $\mathbf{R}_1$  and  $\mathbf{R}_2$  are the centers of mass of the nucleons in the first and second clusters.

We express the wave function of the initial  $(A+p)$  state as follows:

$$\Psi_i = e^{ik_0 r_0} \Psi_A(J_i M_{J_i} T_i M_{T_i}), \quad (2)$$

where  $e^{ik_0 r_0}$  is wave function of the incident proton.

For the final  $(pX+A-X)$  state of wave function, we have

$$\Psi_f = e^{ikr} e^{ik_{pX} r_{pX}} e^{iq' R_{A-X,X}} \varphi(J_X M_{J_X} T_X M_{T_X}) \phi(J_{A-X} M_{J_{A-X}} T_{A-X} M_{T_{A-X}}). \quad (3)$$

Here  $\varphi(J_X M_{J_X} T_X M_{T_X})$  the internal wave function of the association  $X$ ,  $\phi(J_f M_{J_f} T_f M_{T_f})$  the wave function of the final nucleus  $A-X$ , and  $R_{A-X,X}$  the coordinates of the centers of mass of these systems,  $q'$  the momentum of the relative motion of association  $X$  and the final nucleus.

The wave functions (2) and (3) have the wrong asymptotic at large distances, but a good description of the elastic form factors. In the plane wave approximation the matrix element of the reaction has the form

$$F_{if} = (\Psi_f, T_{pX} \Psi_i), \quad (4)$$

$$T_{pX} = V_{pX} + V_{pX} \frac{1}{E_i - E_{kin.} + U + i\eta} T_{pX}, \quad (5)$$

where  $E_i = p_i^2 / 2m_p - \varepsilon_{cs}$  ( $\varepsilon_{cs}$  binding energy association in the initial nucleus  $A$ );  $T_{pX}$  the total kinetic energy of the fly out nucleon and particles of the association.  $V_{pX}$  the potential interaction of the proton with all nucleons association  $X$ ;  $U$  potential, determines the interaction with other nucleons  $X$  nucleons:

$$U = V_N + V_{ls}, \quad (6)$$

where  $V_N$  optical potential, and  $V_{ls}$  –spin-orbital interaction:

$$V_{ls} = V_1 \frac{dV_N}{dr} (\vec{l} \vec{s}). \quad (7)$$

The radial part of the spin-orbit interaction is not well known. However, the experimental data and theoretical considerations suggest that, since the orbital angular momentum  $L$  depends on the distance between the nucleon and the center of nucleus, this function must take the highest value on the surface of the nucleus, and must be small inside the nucleus.

Since we are considering the area of the incident particle energy is less than the threshold for mesons we will use the impulse approximation. The operator  $T$  is introduced by the relation

$$T_{pX} \Psi_i = V_{pX} \Psi_i. \quad (8)$$

Selecting the potential  $V_{pX}$  as the Gaussian potential form:

$$V(r) = -V_0 e^{-r_0^2/r^2}, \quad (9)$$

where  $r_0$  is length of Compton wave of the nucleon,  $V_0$  the depth of the potential. We can determine the matrix (5) in impulse approximation

$$F_{if} = (2\pi\hbar)^{1/2} F^0 R_{LM_L}(q) \left\langle a-X, J_f M_{J_f} T_{T_f} X J_X M_X T_X \mid e^{iq(r_{A-X} - r_X)}; A J_i M_i T_i \right\rangle, \quad (10)$$

here  $F^0$  is the amplitude of the scattering of free proton and association in the center of mass of particles

$$F^0 = \left( e^{iQr} \varphi_p \varphi_X T_{pX} e^{iQ'r'} \varphi'_p \varphi'_X \right), \quad (11)$$

where  $Q$  and  $Q'$  are wave vectors of center of mass and  $R_{LM_L}(q)$  - radial function of center of mass of association. The partial amplitude of invariant SU(4) conversion and the spin-isospin turning of association occur without recharging.

The cross section of the reaction  $A(p, pX)A-X$  in the impulse approximation is as the following:

$$\frac{d\sigma}{d\Omega} = 16\pi m_i m_f |F^0|^2 R_{LM_L}(q)^2 \delta(k_0 - k_p - p_X - p_q) \delta(E_i - E_f) \sum_{J_f J_i} \theta_{J_i}^{A,p} \theta_{J_f}^{pX,A-X}, \quad (12)$$

$$x \left\langle A-X, J_f M_{J_f} T_{T_f} X J_X M_X T_X \mid e^{iq(r_{A-X} - r_X)}; A J_i M_i T_i \right\rangle^2$$

where  $m_i = \frac{M_A m_p}{M_A + m_p}$ ,  $m_f = \frac{M_{A-X} m_{pX}}{M_{A-X} + m_{pX}}$ ,  $\theta_{J_i}^{A,p}$  and  $\theta_{J_f}^{pX,A-X}$  is reduced width of the corresponding channel in units of Wigener limit.

The main feature of the integrals appearing in the expressions (10) and (12), is that they are presented in nucleon variables. Correlations, in particular cluster formation, have to be considered in low-density nuclear systems. A possible approach is the introduction of clusters as effective degrees of freedom in the model description. The full antisymmetrization (Pauli blocking) is indispensable at increasing densities. Equation (11) for the cross section of the associative knocking out nucleons obtained in the impulse approximation and scattering matrix is independent of recoil momentum. If we assume that the scattering cross section depends on  $q$ , then (12) is not correct. Included in it  $F^0$  must be renormalized, considering dependence the cross section on  $q$ .

### 3. Conclusions

Problem in solving such problems is the separation of variables, because it affects the possibility of analytical calculation of integrals over the angular variables, as well as for those variables that are not associated with the interaction of particles. The classification of states according to the orbital schemes of Jung, which automatically leads to orthogonal functions with different partitions of associations.

The angular distribution of the scattering of composite particles, including associations is equally well described by different sets of optical potential. The experimental results clearly demonstrate the benefit of the presence of spin-orbit term in the optical potential. But the intensity

obtained from inelastic reactions and transmission, is much greater than would be expected based on the convolution with a realistic nucleon-nucleon interaction. But unambiguous definition of optical model of potential parameters only from the analysis of the angular distributions of the elastic scattering, in principle, is not feasible because the elastic scattering of complex particles is determined not only potential scattering. A significant contribution to the angular distribution, especially in the region of large angles may make other reaction mechanisms, in particular exchange processes.

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## ПОТЕНЦИАЛЬНАЯ МОДЕЛЬ ВЗАИМОДЕЙСТВИЯ НУКЛОННЫХ АССОЦИАЦИЙ

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**Резюме:** Кластерная модель применяется для изучения ассоциативного выбивания нуклонов в процессах  $A(p,pX)A-X$ . Исследование проводится в импульсном приближении с плоскими волнами с учетом потенциала Гаусса. Предлагается, что матрица рассеяния не зависит от передаваемого импульса. Парциальные амплитуды инварианты  $SU(4)$  преобразованиям. Внутренней структурой ассоциации пренебрегается и эффекты принципа Паули между нуклонами в ассоциации учитываются путем введения короткого отталкивания между кластерами. Спин-изоспиновый переворот ассоциации к перезарядку не приводит. Классификация состояний ассоциаций по орбитальным схемам Юнга автоматически приводит к ортогональности волновых функций с разными разбиения ассоциаций.

**Ключевые слова:** нуклонная ассоциация, рассеяние, импульсное приближение, спин - орбитальное взаимодействия, потенциал Гаусса.

## NUKLON ASSOSIASİYALARININ POTENSİAL QARŞILIQLI TƏSİRLƏRİ

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**Xülasə:** Klaster modeli  $A(p,pX)A-X$  proseslərində nuklon assosiasiyalarının vurulub çıxarılmasının öyrənilməsinə tətbiq edilir. Tədqiqat impuls yaxınlaşmasında müstəvi dalğalarla Qauss potensialı vasitəsilə aparılır. Səpilmə matrisasının ötürülən impulsdan asılı olmaması qəbul edilmişdir. Parsial amplitudlar  $SU(4)$  çevrilməsinə görə invariantdır. Assosiasiyaların daxili strukturu nəzərə alınmamışdır və onların nuklonları arasında Pauli prinsipindən yaranan effektlər nuklonlar arasında qısa dəfətmənin daxil edilməsi ilə nəzərə alınır. Assosiasiyaların spin-izospin fırlanması yüklənməyə gətirmir. Yünqun orbital sxemi assosiasiyaların müxtəlif bölünməsinə uyğun hal funksiyalarının ortogonallığına gətirir.

**Açar sözlər:** nuklon assosiasiyası, səpilmə, impuls yaxınlaşması, spin-orbital qarşılıqlı təsir, Qauss potensialı.