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DEFINITION OF REFRACTIVE INDEX OF NEUTRON WAVES IN CRYSTALS

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Abstract: To describe the scattering of neutron wave by the density fluctuations of the crystal it has been applied the optical potential. The discussion applies to the case where there is only one isotope of the element. Studies of analytic properties of the scattering amplitude allow translating the results of the axiomatic study of scattering by simple physical language.These formulas have been obtained under the assumption that the imaginary part of optical potential is a local operator. The results are applied to the calculation of refractive index of the neutron wave. It was determined that the scattering in density fluctuations does not contribute to attenuation of the coherent neutron wave.

Keywords: Scattering, cross section, refractive index, fluctuation density.

1. Introduction

Nuclear processes with low energy neutrons make up the most-studied field of physics of nuclear interactions. Made substantial progress here is associated with the emergence of high-quality experimental information on the energy and angular dependence of the corresponding sections and with the apparent success of the theoretical interpretation of the data. The presence of intense thermal neutron fluxes from stationary reactors allows to measure small cross sections [1]. But it should be noted that the possibility of obtaining full information in studies of low-energy neutrons are limited, and there is not a full understanding of the mechanism of interaction of neutrons with matter.

Passing through matter, neutrons cause various nuclear reactions and elastically scattered by nuclei. The intensity of these microscopic processes ultimately determines all the macroscopic properties of matter, such as slowing, diffusion, absorption, and etc. Since the neutron has zero electrical charge, it almost does not interact with the electrons of the atomic shells. Therefore, the atomic characteristics of the medium do not play any role in the spread of neutrons in matter. This is purely a nuclear process.

2. Passage of the neutron wave through a matter

The process when neutron are scattered by matter, can change the momentum and energy of the neutrons. In a crystalline substance atoms are arranged in an orderly manner in space. Neutron waves add up the point of observation in accordance with the laws of interference if the phase difference between the scattered waves is constant (coherent scattering), we can observe the pattern of alternating in the space diffraction minima and maxima. If the order in the arrangement of atoms is broken, scattering will not be coherent.

Consider the passage of neutrons through a crystal. The discussion applies to the case where there is only one isotope of the element.

Refractive indices for neutron is close to unity and difficult to measure its. If the plate has a thickness d and refractive index n the neutron wave undergoes a phase shift and exits in the form of

$$\psi_k(z > d) = e^{ik(z-d)+nkd} . \tag{1}$$

Here k momentum of neutron after scattering.

Now suppose that the plate substance contains of N localized impurity - scatters and these centres scatters spherically symmetric wave with the scattering length a_l

$$a_l = \frac{1}{kctg\delta_l} = \begin{cases} (\delta_l - \pi)/k & \text{for} \quad |\delta_l - \pi| << 1, \\ \delta_l / k & \text{for} \quad |\delta_l| << 1. \end{cases} \tag{2}$$

where δ_l scattering phase.

Then, for the $z>d$ the wave can be expressed as

$$\psi_k(z > d) = e^{ikz} - a_l N d 2\pi \int r dr \frac{e^{ikr}}{r} \cdot \tag{3}$$

The integral can be easily calculated using the coefficient convergence $\exp(-\omega r) \quad (\omega > 0)$

$$\int_z^\infty e^{ikr} dr = \lim_{\omega \to 0} \int_z^\infty e^{(ik-\omega)r} dr = -\frac{e^{ikz}}{ik} \ , \tag{4}$$

so that

$$\psi_k(z > d) = e^{ikz} - 2\pi a_l N d \frac{e^{ikz}}{k^2} \tag{5}$$

If (1) submit in the form

$$\psi_k(z > d) \approx e^{ikz}\left[1 + ik(n-1)d\right] \tag{6}$$

and compare with (5), we obtain

$$n - 1 = -2\pi a_l N \frac{1}{k^2} \tag{7}$$

If the scattering is not spherically symmetric, which corresponds to the amplitude $f\ (\theta)$, we obtain:

$$n - 1 = -2\pi N \frac{1}{k^2} f(0) \, , \tag{8}$$

where $f(0)$ forward scattering amplitude because the refractive index describes the propagation of waves in the forward direction.

Consider the effect of the inhomogeneity of the crystal with the volume V on the distribution of coherent neutron wave. Inhomogeneity can be caused by dynamic density fluctuations, and be statistical in nature. Fluctuations in the density of the scattering material cause neutron scattering wave.

We define the scattering cross-section of the fluctuation of the N localized impurity - scatterers. In those cases where the neutron wavelength is large compared to the size of the impurity following model can be used [2]

$$\delta\eta(r) = \sum_{i=1}^{N} \delta\eta_0(r - R_i) \, , \tag{9}$$

where $\delta\eta$ - random density fluctuations, R_i the radius vector of the centre of gravity of i impurity, and $\delta\eta_0$ describes the action of one of the scattering centre.

Neutron scattering wave can be taken into account by choosing the real part of the optical potential in the form of:

$$\delta U_R = (2\pi\hbar^2 / \mu) < \delta\eta > \mathrm{Re}\, f(0). \qquad (10)$$

Here brackets <...> denote averaging over the distribution of static states of the scattering system. Averaging over configurations of the scattering nuclei, ie, at the equilibrium position is an independent operation only in case of the crystal.

Similar to (10), can select the imaginary part of the optical potential

$$\delta U_I = (2\pi\hbar^2 / \mu) < \delta\eta > \mathrm{Im}\, f(0). \qquad (11)$$

The imaginary part of the potential is models the inelastic processes related to elastic scattering and determines the weakening of the coherent wave in the entrance channel. In general, optical potential is nonlocal and depends of neutron's energy. For thermal neutron the dependence of energy and nonlocalness of the optical potential is very little effect on the propagation of a neutron wave in the crystal.

According to the optical theorem

$$\mathrm{Im}\, f(0) = -k(\sigma_{abs} + \sigma_{inel}) / 4\pi \qquad (12)$$

where referring to [4]

$$\sigma_{abs.} = NV\left\langle (\delta\eta_0)^2 \right\rangle, \qquad (13)$$

$$\sigma_{inel.} = N\sigma_{el.} N\, V <(\delta\eta)^2 >, \qquad (14)$$

σ_{abs} cross section of absorption and σ_{inel} cross section of inelastic scattering. Elastic scattering related unitarily condition with all inelastic processes. Since the cross section of inelastic scattering in the approximation of a heavy target is proportional to the cross section of elastic scattering [3].

Elastic scattering cross section is equal to

$$\sigma_{el.}(k) = \int |\, f(k,\theta)|^2 d\Omega, \qquad (15)$$

and expression (15) can be represented as a series expansion in the multiplicity of elastic scattering

$$\sigma_{el.}(k) = N_1 \int |\, f(k,\theta)|^2 d\Omega + \frac{N_2}{k^2} \int dk' |\, f(k') f(k-k')|^2 d\Omega + ..., \qquad (16)$$

where

$$Ni = \frac{1}{\sigma_1 i!} \int \exp(-\sigma_1 \delta\eta\, r)^i\, dr. \qquad (17)$$

Here σ_1 a total scattering cross section, related to the scattering center:

$$\sigma_1 = 4\pi \frac{V}{V_0} \left\langle \left(\frac{\delta\eta}{\eta_0} \right)^2 \right\rangle \qquad (18)$$

where V_0 - the volume per one scattering nucleus, and *i*- the number of scattering nuclei per unit volume of the crystal.

These cross sections (13), (14) and (15) describes the processes in which the number of particles in the scattering system remains the same, namely, elastic scattering, the scattering of particles with excitation and scattering the scattering system, accompanied by partial or total decay of the scattering system.

Optical theorem relates the refractive index of substance with the scattering cross section of individual atoms and nuclei of which consist the material.

If the refractive index contains an imaginary part, it is necessary to divided (8) into two parts: real and imaginary:

$$\mathrm{Re}(n-1)=\frac{2\pi N}{k^2}\mathrm{Re}\, f_\kappa(0); \qquad \mathrm{Im}\, n=\frac{2\pi N}{k^2}\mathrm{Im}\, f_\kappa(0) \tag{19}$$

Imaginary part of the refractive index determined by the condition of the decreasing the particle in the channel of elastic scattering.

Determining the effective cross section by the optical theorem (12) for the imaginary part of the refractive index obtain the following expression

$$\mathrm{Im}\, n=\frac{2\pi N^2}{k}\left\langle\left(\frac{\delta\eta}{\eta_0}\right)^2\right\rangle. \tag{20}$$

At $kf_k(0)<<1$ the imaginary part of the scattering amplitude is small and the refractive index can be considered real. Thus, in this case, scattering in density fluctuations does not contribute to the attenuation of the coherent neutron wave.

Knowing the effective wave number of the neutron wave in the medium and the refractive index can be calculated reflection and transmission coefficients for the neutron wave for the finite-volume substances.

3. Conclusion

These formulas have been obtained under the assumption that the imaginary part of the optical potential is a local operator. If do not resort to this hypothesis, have to deal with the time-consuming calculation of the sum of the vectors of the crystal lattice.

The above discussion applies to the case where there is only one isotope of one element present (especially an element with zero nuclear spin), however practically all real systems will have a distribution of both elements and isotopes of those elements. Moreover for thermal neutrons due to the shallow depth of penetration into the wall of the crystal must carefully consider the effect of the surface structure. The crystal surface is two-dimensional defect, distorting the frequency spectrum of vibrations of atoms located in the surface layer of the lattice.

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ОПРЕДЕЛЕНИЕ ПОКАЗАТЕЛЯ ПРЕЛОМЛЕНИЯ НЕЙТРОННЫХ ВОЛН В КРИСТАЛЛАХ

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Резюме: Для описания рассеяния нейтронных волн флуктуацией плотности кристалла применяется оптический потенциал. Исследование проводится для случая, когда элемент имеет только один изотоп. Изучение аналитических свойств амплитуды рассеяния позволяют перевести результаты аксиоматического изучения рассеяния с помощью простого физического языка. Формулы были получены в предположении, что мнимая часть оптического потенциала является локальным оператором. Результаты применяются для вычисления показателя преломления нейтронной волны. Было определено, что рассеяние на флуктуациях плотности не способствует ослаблению когерентной нейтронной волны.

Ключевые слова: Рассеяние, поперечное сечение, показатель преломления, флуктуация плотности.

KRİSTALLARDA NEYTRON DALĞALARININ SINDIRMA ƏMSALININ TƏYİNİ

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Xülasə: Neytron dalğalarının səpilməsini kristalın sıxlıq fluktuasiyası ilə təsvir etmək üçün optik potensial tətbiq olunmuşdur. Tədqiqat elementin yalnız bir izotopa malik olduğu hala aid edilir. Səpilmə amplitudasının analitik xüsusiyyətlərinin öyrənilməsi səpilmənin aksiomatik tədqiqinin nəticələrini sadə fiziki dillə çevrilməsinə imkan verir. Formulalar optik potensialın xəyali hissəsinin lokal operator olması yaxınlaşmasında alınmışdır. Nəticələr neyton dalğasının sındırma əmsalının hesablanmasında tətbiq olunur. Müəyyən edilmişdir ki, sıxlıq fluktuasiyalarından səpilmələr koherent neytron dalğalarının zəifləməsinə gətirib çıxarmır.

Açar sözlər: Səpilmə, en kəsiyi, sındırma əmsalı, sıxlıq fluktuasiyası.