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THE INVESTIGATION OF GIANT RESONANCES IN NUCLEI BY INELASTIC SCATTERING OF PROTONS

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Abstract: The obtained form-factor for inelastic scattering considering energy losses is expressed as a function of plane-wave approximation, on the basis of nonrelativistic scattering of nucleons in nuclei at the high-energy distorted-wave approximation in three-dimensional form. By calculating the double differential cross section of scattering, the energy losses of scattered protons with incident energy of 800 MeV were defined. The giant dipole and quadruple resonance with the vibration surface of nuclear ^{208}Pb were investigated.

Keywords: Nuclear reactions, $^{208}Pb(p,p')$ 800 MeV, vibration parameters, quadrupole moment.

1. Introduction

The increased interest to studies the structure of nuclei by elastic and Inelastic scattering of nucleons is due to the numerous precise experimental data on a number of nuclei at different energies and large transferred momentum [1-3]. By getting a simple and accurate analytical expression for the scattering amplitude of the processes, a lot of important information about the structure of nuclei can be drawn from these data [4-6].

2. The proposed theory

To study the properties of highly excited states of nuclei by inelastic scattering of protons, considering the energy loss we use the scattering amplitude obtained in analytical form by the method of distorted waves in [7].

We write the differential cross section in the form

$$\frac{d\sigma_{if}}{d\Omega}=\frac{k_f}{k_i}\frac{2J_f+1}{2J_i+1}\sum_{LM}\frac{\left|F_{LM}(q)\right|^2}{2L+1}\frac{\Gamma_L^2/4}{(E_x-E_L)^2+\Gamma_L^2/4} \quad (1)$$

Resonant excitation is considered here in Breit – Wigner form, where the energy loss is equal to the energy difference between the incident and scattered particles $E_x=E_i-E_f$.

The nuclear form factor $F_{LM}(\mathbf{q})$ obtained in [8] has the following form:

$$F_{LM}(\mathbf{q})=\frac{ik\sigma_{NN}(1-i\varepsilon_0)\ell^{\frac{-\beta_0^2q^2}{2}}}{4\pi}\int \ell^{i\mathbf{qr}}\Re(\mathbf{r})\rho_L(r)Y_{LM}^*(\hat{r})d\mathbf{r}\ , \quad (2)$$

where

$$\Re(\mathbf{r})=1+i\phi(\mathbf{r})-\beta_0^2\mathbf{q}\nabla\phi(\mathbf{r}) \quad (3)$$

Function $\phi(\mathbf{r})$, arising on account of the distortions in the incident and outgoing waves, has the form:

$$\phi(\mathbf{r}) = -\frac{U(0)}{2E_i}\mathbf{qr} - \frac{a}{12}\left(3k^2r^2(\mathbf{k}_i\mathbf{r} - \mathbf{k}_f\mathbf{r}) + 2(\mathbf{k}_i\mathbf{r})^3 - 2(\mathbf{k}_f\mathbf{r})^3\right) - \\ -b\left([\mathbf{rk}_i]^2 + [\mathbf{rk}_f]^2\right) + c\left([\mathbf{rk}_i]^4 + [\mathbf{rk}_f]^4\right) \quad (4)$$

Explicit expressions of potential in the center of the nuclei - $U(0)$, the parameters a, b and c, depending on the density of nucleon distribution in nuclei, as well as a parameter β_0^2 (the slope of the diffraction peak), which is a part of the amplitude of the free NN – interaction, are given in [8].

To calculate the integral (2) we choose a coordinate system in which $Oz \uparrow\uparrow \mathbf{q}$ and designating $\cos\hat{q}\hat{r} = \mu$, the impulse transmitted to the core (considering energy loss of the incident proton) is written as:

$$|\mathbf{q}| = |\mathbf{k}_i - \mathbf{k}_f| = \sqrt{k_i^2 + k_f^2 - 2k_ik_f\cos\vartheta} = \left(\frac{2m}{\hbar^2}\right)^{1/2}\sqrt{E_i + E_f - 2E_i^{1/2}E_f^{1/2}\cos\vartheta} \quad (5)$$

Angle of scattering $\vartheta = \vartheta_1 + \vartheta_2$ and deflection angles of the incident (ϑ_1) and the scattered particles (ϑ_2) relatively ox -axis in three-dimensional coordinate system (Fig.1) are related as follows:

$$\cos(\hat{\mathbf{r}},\hat{\mathbf{k}}_i) = \sqrt{1-\mu^2}\cos\vartheta_1\cos\varphi + \mu\sin\vartheta_1, \\ \cos(\hat{\mathbf{r}},\hat{\mathbf{k}}_f) = \sqrt{1-\mu^2}\cos\vartheta_2\cos\varphi - \mu\sin\vartheta_2 \quad (6)$$

$$tg\,\vartheta_2 = \frac{E_f^{1/2}}{E_i^{1/2}}\frac{1}{\sin\vartheta} - ctg\,\vartheta \quad (7)$$

After integrating over the angles, the form factor is reduced to one-dimensional integral

$$\tilde{F}_L(q) = \frac{2\pi i}{k_i}F_L(q), \quad (8)$$

where

$$F_L(q) = \sum_{\varepsilon=\pm1}\varepsilon\int_0^\infty \Re(\varepsilon r)G_L(r)\ell^{i\varepsilon qr}\rho_L(r)rdr \quad , \quad (9)$$

$$\Re(\varepsilon r) = \Re_0(q) + \varepsilon\Re_1(q)\cdot r + \Re_2(q)\cdot r^2 + i\varepsilon\Re_3(q)\cdot r^3 \quad , \quad (10)$$

$$\Re_0(q) = 1 + \frac{U(0)}{2E_i}\beta_0^2q^2, \quad (11)$$

$$\Re_1(q) = -\frac{U(0)}{2E_i}q + 2\beta_0^2qb(k_i^2\cos^2\vartheta_1 + k_f^2\cos^2\vartheta_2) \quad (12)$$

$$\Re_2(q) = -ib(k_i^2\cos^2\vartheta_1 + k_f^2\cos^2\vartheta_2) - + \frac{ak_i}{4E_i}\beta_0^2 q(3k_i^2 q + 2k_i^3\sin^3\vartheta_1 + 2k_f^3\sin^3\vartheta_2) \quad (13)$$

$$\Re_3(q) = -\frac{ak_i}{12E_i}(3k_i^2 q + 2k_i^3\sin^3\vartheta_1 + 2k_f^3\sin^3\vartheta_2) \quad (14)$$

and

$$G_L(r) = \sum_{\nu=0}^{L}\frac{i^\nu}{(qr)^\nu}\left[\frac{\partial^\nu Y_{L0}(\mu)}{\partial\mu^\nu}\right]_{\mu=\varepsilon} \quad (15)$$

The final expression of the differential cross sections for inelastic scattering of nucleons on nuclei we write in the form:

$$\frac{d\sigma_{if}}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{NA}\sum_{L=0}^{\infty}\frac{\left|\tilde{F}_L(q)\right|^2}{2L+1}\frac{\Gamma_L^2/4}{(E_x - E_L)^2 + \Gamma_L^2/4}, \quad (16)$$

where

$$\left(\frac{d\sigma}{d\Omega}\right)_{NA} = \frac{\sigma_{NN}^2 k_i^4(1+\varepsilon_0^2)}{32\pi^2 q^2}\ell^{-\beta_0^2 q^2} \quad (17)$$

- is the cross-section of nucleon - nucleon scattering in nucleus.

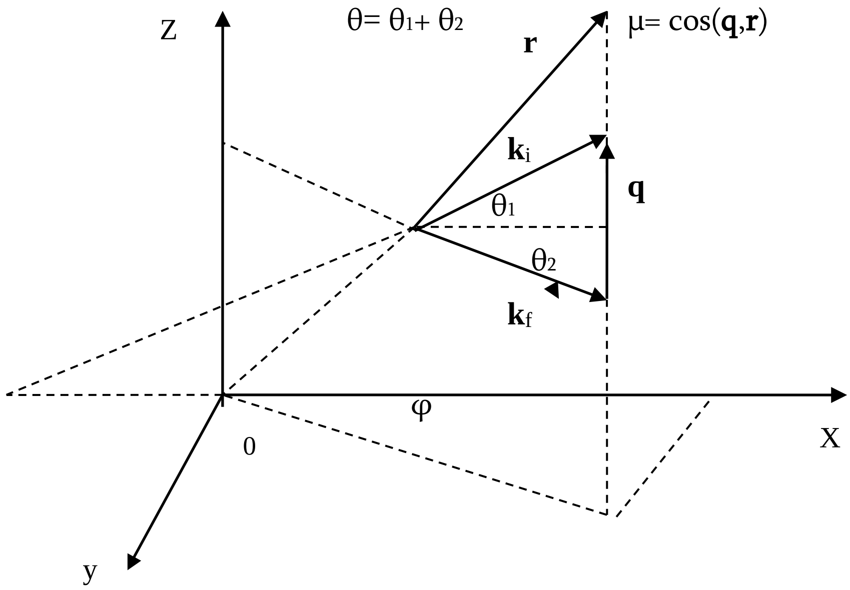

Fig. 1 Impulses of incident ($\mathbf{k}_i$) and scattering ($\mathbf{k}_f$) particles in three- dimensional coordinate system with the transfer impulse $\mathbf{q} = \mathbf{k}_i - \mathbf{k}_f$.

To study the dependence of the cross section on the final energies of the scattered particles, the general expression of the double differential cross section is written:

$$\frac{d^2\sigma_{if}}{d\Omega dE_f} = \left(\frac{d\sigma}{d\Omega}\right)_{NA} \frac{1}{E_i^{1/2} E_f^{1/2}} \sum_{L=0}^{\infty} \frac{\left|\tilde{F}_L(q)\right|^2}{2L+1} \frac{\Gamma_L^2/4}{(E_x - E_L)^2 + \Gamma_L^2/4} \tag{18}$$

3. Application of the theory

Using the quantum hydrodynamic model of the nucleus [8], we investigate the properties of highly excited states of nuclei in the energy region of giant resonances.

In order to consider in the excited nucleus the connection of a giant resonance with the vibrations of the nuclear surface, we combine the low-energy collective degrees of freedom with the high-energy one. The interaction between these motions is very strong, so it can significantly affect the structure of the giant resonances [9]. Let's present the proton density of the excited nucleus as the sum of the equilibrium proton density $\rho_p(\mathbf{r})$ and the density of fluctuations, responsible for the giant resonances $\rho_p(\mathbf{r})\eta^{Gr}(\mathbf{r},t)$ extending from the center to the surface, and vibrations of the nucleus surface $\rho_p(\mathbf{r})\eta^{vib}(\mathbf{r},t)$ as:

$$\rho_p(\mathbf{r},t) = \rho_\rho(\mathbf{r})[1 + \eta^{Gr}(\mathbf{r},t) + \eta^{vib}(\mathbf{r},t)] \tag{19}$$

According to the collective model $\eta^{Gr}(\mathbf{r},t)$ can be written as:

$$\eta^{Gr}(\mathbf{r},t) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} q_{lm}(t) A_l j_l(k_l r) Y_{lm}^* \tag{20}$$

and the of fluctuations density on the surface of the nucleus we present in the form of an expansion in the collective coordinate $-\alpha_{\lambda\mu}(t)$:

$$\eta^{vib}(\mathbf{r},t) = \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu}(t) A_\lambda j_\lambda(k_\lambda r) Y_{\lambda\mu}^* \tag{21}$$

Values A_l are normalization factors which are determined from the normalization conditions:

$$A_l = \sqrt{2} R_0^{-\frac{3}{2}} [j_l^2(k_l r) - j_{l-1}(k_l r) j_{l+1}(k_l r)]^{-\frac{1}{2}} \tag{22}$$

As shown in [3], the potential velocity is a solution of equation

$$\nabla^2 \phi(\mathbf{r}) + k^2 \phi(\mathbf{r}) = 0, \tag{23}$$

which can be represented as an expansion

$$\phi(\mathbf{r},t) = \sum_{\lambda=0}^{\infty} \phi_\lambda(\mathbf{r},t) = \sum_{\lambda\mu} S_{\lambda\mu}(t) j_\lambda(k_\lambda r) Y_{\lambda\mu}^* \tag{24}$$

Collective oscillations of protons density respectively neutrons lead to a harmonically varying deformation of proton substance near the initial spherical equilibrium shape ($r' = R_0$)

$$r(\theta,\varphi,t) = r'\{1 + \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu}(t)(\frac{r'}{R_0})^\lambda Y_{\lambda\mu}^*(\theta,\varphi)\} \tag{25}$$

Using condition

$$\frac{dr(\theta,\varphi,t)}{dt}\Big|_{r'=R_0}=\frac{\partial\phi(\theta,\varphi,t)}{\partial r}\Big|_{r'=R_0}\quad , \tag{26}$$

we find the relation between the coefficients $S_{\lambda\mu}(t)$ of the expansion the potential of velocities and coefficients $\alpha_{\lambda\mu}(t)$, which define the shape of the density distribution of the nucleons on the surface of the nucleus

$$S_{\lambda\mu}(t)=\frac{\dot{\alpha}_{\lambda\mu}(t)R_0}{\nabla_r\, j_\lambda(k_\lambda r)\big|_{r=R_0}} \tag{27}$$

Nucleon density distribution in the ground state of the nucleus $\rho(r)$ we choose in the form of the Fermi - functions:

$$\rho_N(r)=\rho_0(1+\ell^{\frac{r-R_0}{z}})^{-1} \tag{28}$$

To reveal the surface effect, the equilibrium density is expressed in the form of two terms [10]:

$$\rho_p(r)=\rho_{0p}S(r-R_0)-\rho_{0p}\frac{\pi^2z^2}{6}\delta'(r-R_0) \tag{29}$$

where S - a step function, $S=1$ with $r<R_0$ and $S=0$ under $r>R_0$, and δ'-derivative of δ -function.

Low-energy spectra of spherical nuclei are often the typical spectra of almost harmonic surface vibrations quadruple type [9]. Therefore, for the expression (21), limited by the term $\lambda=2$ for the transition density, describing the quadruple oscillations of the nuclear surface, we obtain:

$$\rho_2=A_2\sqrt{\frac{\hbar}{2B_2^{vib}\varpi_2}}\,j_2(k_2r)\rho_p(r) \tag{30}$$

and for the so-called mass parameter B_λ in general form, we get

$$B_\lambda^{vib}=\frac{m\rho_0k_\lambda^2\pi^2z^2R_0^2}{6[\nabla j_\lambda(k_\lambda r)\big|_{r=R_0}]^2}\frac{\partial}{\partial r}[r^2j_\lambda^2(k_\lambda r)]_{r=R_0}, \tag{31}$$

At that the excitation energy is:

$$\hbar\varpi_\lambda=\hbar\sqrt{\frac{C_\lambda^{vib}}{B_\lambda^{vib}}} \tag{32}$$

Here, the stiffness coefficient is determined by the expression

$$C_\lambda^{vib}=(\lambda-1)(\lambda+2)R_0^2\sigma-\frac{\pi(Ze)^2z^2}{8(2\lambda+1)R_0^3}\quad , \tag{33}$$

where σ - coefficient of surface tension of the core [9].

Thus, after integrating (9) for the form factor responsible for the quadruple oscillations of the nuclear surface, we have

$$F_{\lambda=2}(q)=\sqrt{\frac{\hbar}{2B_2^{vib}\varpi_2}}\frac{\pi^2 z^2\rho_{0p}A_2}{6}F'(q)\ , \tag{34}$$

where

$$F'(q)=\sum_{\varepsilon=\pm1}\varepsilon\frac{\partial}{\partial r}[r\Re(r)\,j_2(k_2r)G_2(r\varepsilon)\ell^{i\varepsilon qr}]_{r=R_0} \tag{35}$$

Then the form-factor responsible for the giant resonances takes the form

$$F_L(q)=\rho_{0p}A_L\sqrt{\frac{\hbar}{2B_L\Omega_L}}\sum_{\varepsilon=\pm1}\varepsilon\int_0^{R_0}r\Re(r)G_L(r\varepsilon)\,j_L(k_Lr)\ell^{i\varepsilon qr}dr \tag{36}$$

4. Results and discussions

This theory has been applied to the inelastic scattering of protons with incident energy 800 MeV on nucleus ^{208}Pb. In this case for the nuclear radius and the thickness of the surface layer characterizing the distribution nucleons in the ground state, we used the values derived from the elastic scattering of electrons ($R_0 = 6{,}28\ Fm$, $z = 0{,}314\ Fm$).

Comparison of the obtained results for the double-section in the energy dependence of the scattered protons with the experimental [11] and theoretical data obtained by Glauber approximation method [12], at scattering angle $\vartheta = 13^\circ$, are shown in Fig. 2.

At this scattering angle the energy loss of incident protons is ~ 45 MeV. This means that at this angle of scattering and these energy losses of incident protons the giant dipole and quadruple resonances with energies and widths of excitation $\hbar\Omega_1 = 13{,}18$ MeV and $\Gamma_1 = 2{,}3$ MeV; $\hbar\Omega_2 = 21{,}18$ MeV and $\Gamma_2 = 4{,}1$ MeV, as well as the vibration of the surface of the nucleus with energy - $\hbar\omega_2 = 4{,}09$ MeV and width $\Gamma_{\lambda=2} = 0{,}23$ MeV may appear in the nuclei.

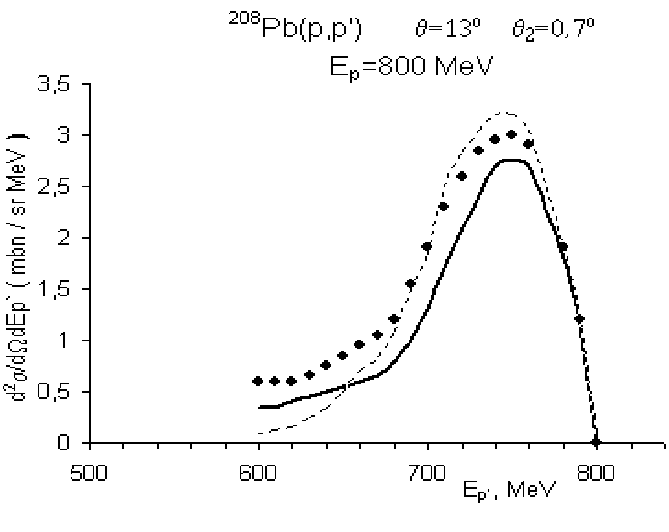

Fig.2 Dependence of the double differential cross section at energy of incident protons 800 MeV, scattering angle $\vartheta = 13^\circ$ for nuclear $^{208}Pb(p,p')$ on the energy of the scattered protons, solid line-the obtained results, points - experimental data [9], and a bar line – the theoretical calculations obtained in [12].

The parameter characterizing the mean-square deformation of the excited nucleus is determined by the expression

$$\beta = \sqrt{<0\left|\sum_{\mu} \alpha^{*}_{2\mu}\alpha_{2\mu}\right|0>} = \sqrt{\frac{5\hbar\varpi_2}{2C_2^{vib}}} \quad , \qquad (37)$$

for which at the parameter of rigidity $C_2^{vib} = 224\,\text{MeV}$ we get $\beta = 0{,}21$.

As seen from Fig. 2, the calculated theoretical cross section correctly predicts the location of excitation in the nucleus.

However, for all values of the energies of the scattered protons double differential cross section is somewhat underestimated. In addition, the value of the dynamic deformation when compared with obtained from the experiment is a little underestimated, too. Apparently, this is due to the fact that we used nuclear model which is still not quite perfect.

Besides, the calculation of differential cross sections of giant dipole and quadruple excitations was held as well as the cross sections of the quadruple vibrations of the nuclear surface at small scattering angles. The results of calculations in comparison with experimental and theoretical data derived in the distorted-wave Born approximation (DWBA) [12], are shown in Fig. 3.

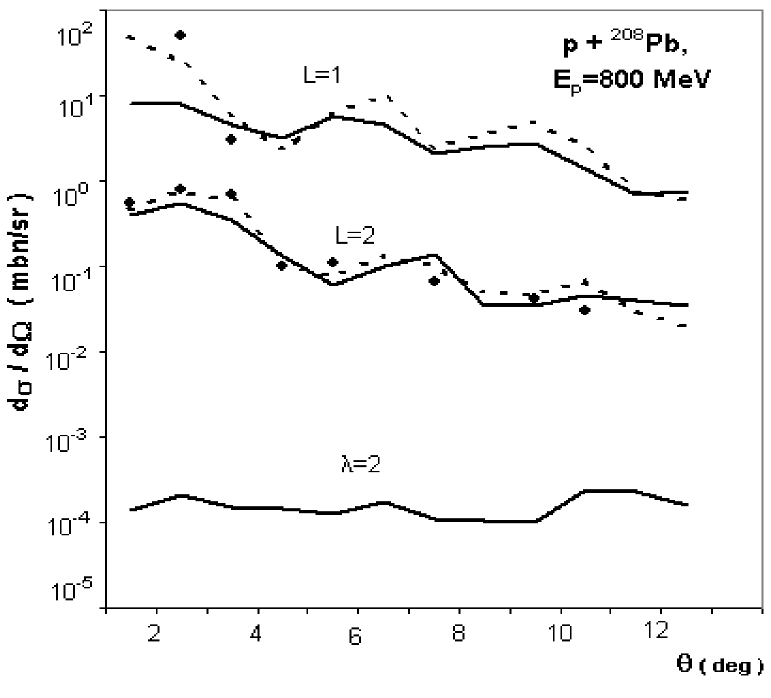

Fig. 3 The dependence of the differential cross section on the scattering angle of protons in the nucleus $^{208}Pb(p,p')$ *at* $E_i = 800$ *MeV for the dipole and quadruple giant resonances, as well as for quadruple vibration of the nuclear surface. Solid line - received results, points - experimental data [7], bar-line - results calculated in DWBA [13].*

As seen from this figure the curves obtained DWBA, and the results of the present work coincide. However, at certain scattering angles corresponding to maxima cross sections, some discrepancy appears.

5. Conclusion

By the method of distorted waves in the analytical form the expression for the amplitude of inelastic scattering of nucleons by nuclei was obtained. Applying this theory of scattering for studying excited states of nuclei we assumed that the vibration of the nuclear surface is a consequence of the decay of giant multiple resonances arising in the center of the nucleus. This allowed expressing in terms of collective coordinates the fluctuations of density and their frequencies on the base of the collective model of the nucleus, the degrees of freedom the quadruple deformation of the surface of the nucleus, described by collective coordinates.

By comparison of the calculated double differential cross sections with the experimental data at scattering angle of protons $\vartheta = 13^{\circ}$ in the nucleus ^{208}Pb, the losses of energy were defined.

The angular dependence of cross sections of highly excited giant dipole and quadruple excitations, as well as low-energy excitations with quadruple surface vibrations of the nucleus was studied.

Analyzing the results we revealed that the expression obtained for the scattering amplitude was very sensitive to the nuclear parameters in the ground state, what'll allow, investigating the important properties of excited nuclei applying more sophisticated nuclear models.

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ИССЛЕДОВАНИЕ ГИГАНТСКИХ РЕЗОНАНСОВ В ЯДРАХ НЕУПРУГИМ РАССЕЯНИЕМ ПРОТОНОВ

М.М. Мирабуталыбов

Резюме: На основе нерелятивистской теории рассеяния протонов на ядрах в искаженно-волновом приближении полученный формфактор для неупругого рассеяния представлен как функционал формфактора плоско-волного приближения. Вычислено двойное дифференциальное сечение для рассеянных протонов с падающей энергией 800 МэВ. Исследованы гигантские дипольные и квадрупольные резонансы с вибрацией поверхности ядра ^{208}Pb.

Ключевые слова: Ядерная реакция $^{208}Pb\ (p,p')$ 800 МэВ, вибрация поверхности ядра, гигантские резонансы.

PROTONLARIN NÜVƏLƏRDƏN QEYRİ-ELASTİKİ SƏPİLMƏSİLƏ NƏHƏNG REZONANSLARIN TƏDQİQİ

M.M. Mirabutalıbov

Xülasə: Qeyri-relyativistik protonların nüvədən qeyri-elastiki səpilməsi üçün təhrif olunmuş dalğalar yaxınlaşmasında alınan formfaktor, müstəvi dalğalar yaxınlaşmasındakı formfaktorun funksialı şəklinə salınmışdır. Enerjisi 800 MeV olan protonların nüvədən səpilməsinin ikiqat diferensial effektiv kəsiyi hesablanmışdır. ^{208}Pb atom nüvəsində nəhəng dipol və kvadrupol rezonanslar və nüvə səthinin vibrasiyası tədqiq edilmişdir.

Açar sözlər: Nüvə reaksiyası, $^{208}Pb\ (p,p')$ 800 MeV, nüvə səthinin vibrasiyası, nəhəng rezonanslar.