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POLARIZATION EFFECTS IN QUADRUPOLE RADIATION

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Abstract: The polarization effects in quadrupole radiation processes have been studied. The unitarity condition was used to determine the scattering amplitudes. The polarization effect essentially depends on the relative value of the behavior of the amplitude. The polarization disappears if one of these terms is zero.

Keywords: quadrupole radiation, cross section, polarization, spin-orbit interaction

1. Introduction

Polarization phenomena take place during the interaction of a particle with a nucleus. The interaction of nuclei with particles at a sufficiently high energy, moving in directions close to parallel to some family of axes, is described by the potential averaged in the direction of these axes. The polarization of an incident particle and the polarization of a recoil particle, as well as the correlation between the polarization directions of these particles, can be determined experimentally. Depending on the direction of polarization, there are many different experimental possibilities.

Although studies with polarized particles and oriented nuclei have been carried out for more than a quarter of a century, it cannot be said that the accumulated experimental material is rich. This situation is due to the great technical difficulties that arise in obtaining polarized particles and oriented nuclei. Here we have to use ultra-low temperatures, very strong magnetic fields, and various other subtle methods. A number of major physical and technical achievements, such as the development of a method for the polarization of slow neutrons by passing through a polarized proton target, the creation of cryostats with the dissolution of ^{3}He and ^{4}He with temperatures $T \approx 10^{-2}$ K and superconducting magnets with fields H$\approx 10^5$E allow us to hope for a significant intensification in the near future of research with polarized particles and oriented nuclei.

In the nuclei the transitions between different states of the L, J, compatible the conservation the total angular momentum. However, the orbital angular momentum and spin are not good quantum numbers and the partial wave corresponding to a certain L and S , may cause scattered wave with $L' \neq L, S' \neq S$. If we take into account that the good quantum numbers are the total angular momentum and parity, the amplitude of the scattered waves form a matrix, the elements of which depend on the following indices: J, L and for quadrupole radiation $\Delta J = J_f - J_i$=2.

A particle with non-zero rest mass and spin J has $2J$+1 quantum state corresponding to different spin orientations (different values of spin projection onto some direction). The particle state is a superposition of these states. If the superposition coefficients are completely determined, then the particle is said to be completely polarized. If the superposition coefficients are not completely determined, but are given only by some statistical characteristics, then one speaks of partial polarization. In particular, the particle may be completely unpolarized; this means that its properties are the same in all directions, like a spinless (with $J = 0$) particle. In the general case, the polarization of particles determines the degree of their symmetry (or asymmetry) in space. Due to the spin dependence of the interaction, the scattering cross section can depend on the polarization of the incident particle and on the polarization of the target.

Semiclassical models are completely inadequate for explaining polarization data, traditional methods such as the distorted wave method and the coupled channel method are successful. The inclusion of higher order terms and links to statistical processes require more work. In connection with pre-equilibrium effects and the emission of fast particles, the transition to the consideration of three-particle channels is inevitable.

In this paper, we consider the polarization function of a scattered nucleon in quadrupole radiation at high energies.

2. Polarization vector

Quadrupole transitions have always been an object of special interest in the study of the structure of nuclei. These processes have been taken on great importance, because they are to be very sensitive to nuclear spectroscopy. In this regard, the number of experimental and theoretical research works [1-6].

Many theoretical and experimental studies have been carried out show that an almost complete polarization of the scattered particles is observed. The polarization effect is a consequence of the fact that the nuclear interaction can be represented as a complex spin-dependent potential. As a result of the spin dependence, the scattering interaction cross section can depend on the polarization of the interacting systems.

Let us consider the calculation of the matrix element for the quadruple transition $0^+ \to 2^+$ in the direct (a, b) reaction in the general case. These reactions can be one-nucleon or two-nucleon direct reactions.

The electric quadrupole moment is a tensor whose components behave like polarizability components, i.e., as a product of two translations. The matrix element of these processes has following expression [7]:

$$M = \frac{\left\langle \Psi_{20} \left| \sum_{m=1}^{A} r_j^2 Y_{20}(\theta,\varphi) \right| \Psi_{00} \right\rangle}{\sqrt{N_i N_j}} \,, \tag{1}$$

where N_i and N_j are normalization integrals,

$$\Psi_{00} = \Psi_{00}(\xi) f(\vec{r}_a) \chi_{1/2,m_s} \,, \tag{2}$$

is the wave function of the initial system: $\Psi_{00}(\xi)$ wave function of the initial nucleus, $f(r_a)$ and $\chi_{1/2,m_s}$ are radial and spin functions, respectively, of the incident particle *a*,
and

$$\Psi_{20} = \Psi_{20}(\xi) f(\vec{r}_b) \chi_{1/2,m_s} \,. \tag{3}$$

is the wave function of the final system: $\Psi_{20}(\xi)$ wave function of the final nucleus, $f(r_b)$ and

$\chi_{1/2,m_s}$ are radial and spin functions, respectively, of the scattered particle *b*.

In the momentum representation the scattering amplitude has the form

$$F(E,\theta)=\frac{1}{(2\pi)^6}\int e^{i(\mathbf{k}_i-\mathbf{k}_j+\mathbf{K})\mathbf{r}_a}e^{-i\mathbf{K}\mathbf{r}_b}\psi_{20}^{*}(\mathrm{k_f}\,\mathrm{K}'|V|\mathrm{k_i}\mathrm{K})\psi_{00}d\mathbf{r}_a d\mathbf{r}_b d\mathbf{K}d\mathbf{K}', \quad (4)$$

where

$$\left(k_f K'|V|k_i K\right)=(2\pi)^3\delta(k_f+K'-k_i-K)\int e^{[i(\mathbf{K}\;-\mathbf{k}_i)\mathbf{r}/2]}V(r)\varphi_{k_i-K}(r)dr. \quad (5)$$

Here $\mathbf{k}_i$ and $\mathbf{K}$ are initial impulses; $\mathbf{k}_f$ and $\mathbf{K}'$ are final impulses.

The potential *V* will be chosen in the following form:

$$V=V_{av}+V_{pair}+V_Q. \quad (6)$$

In (6) V_{av} - Saxon-Woods potential, describes the average field, V_{pair} - residual pair and - V_Q long-range residual quadrupole-quadrupole interaction. Long-range V_Q potentials in deformed nuclei lead to intense interaction between pairs of quasiparticles in states $K^{\pi}=0^+,2^+,\ldots$. As a result, strongly itinerant low-lying states with corresponding quantum numbers appear.

The scattering amplitude $F(E,\theta)$ is elements of the 2 x 2 matrix, which can be expressed in terms of the Pauli matrices $\boldsymbol{\sigma}$ and the unit matrix ***n*** of a unit–amplitude plane wave on the scattered is called the scattering amplitude. In scattering of scalar waves described, in particular, by the Schrödinger equation, the amplitude $F(E,\theta)$ is a scalar [8]:

$$F(E,\theta)=A(E,\theta)+B(E,\theta)(\mathbf{n}\boldsymbol{\sigma}). \quad (7)$$

This expression is valid in the entire range of scattering angles and energy and can be seen as a generalization of the eikonal approximation for the scattering amplitude.

In (7), the first term corresponds to the interaction that does not depend on the spin, and the second term corresponds to the interaction that causes spin reorientation. The explicit form of the functions $A(E,\theta)$ and $B(E,\theta)$ is determined by the specifics of the interaction.

Taking into account the spin-orbit interaction, in the Born approximation, in the momentum representation of $A(E,\theta)$ and $B(E,\theta)$ has the following form:

$$A(E,\theta)=\frac{i}{2}\frac{\hbar^2k^2}{\mu^2c^2}\sin\theta\int dr\exp[-i(\mathbf{k}-\mathbf{k}')\mathbf{r}]\,V(r), \quad (8)$$

$$B(E,\theta)=-\frac{ik^2\hbar^2}{2\pi\mu^2c^2}\int dr\exp[i(\mathbf{k}-\mathbf{k}')\mathbf{r}]Y(r,\theta)\frac{1}{r}\frac{dV}{dr}dr. \quad (9)$$

The differential scattering cross section is given by the formula

$$\frac{d\sigma}{d\Omega}=|<\Psi_{20f}^{*}|F(E,\theta)|\Psi_{00}>|^2=|A(E,\theta)+<\Psi_{20}^{*}|(\mathbf{n}\boldsymbol{\sigma})|\Psi_{00}>|^2. \quad (10)$$

In this case, to calculate the polarization vector **P** of scattered particles, it is necessary to average the vector **P** taking into account the equal statistical weights of different projections of the spin in the initial state:

$$\mathbf{P}^{(0)} = \frac{2\,\mathrm{Re}(A^{*}(E,\theta)B(E,\theta))}{|A(E,\theta)|^{2} + |B(E,\theta)|^{2}}\mathbf{n}\,. \tag{11}$$

So, as a result of the scattering of an initially unpolarized particle with spin s = 1/2 on spinless nucleus, generally speaking, the particle are polarized. It depends on the particle energy E and the scattering angle θ. In this case, the polarization vector **P** is always directed perpendicular to the scattering plane, and its absolute value depends significantly on the value of $\mathrm{Re}\,(A^{*}(E,\theta)B(E,\theta))$. The polarization effect essentially depends on the relative phase of the amplitudes $A(E,\theta)$ and $B(E,\theta)$ and vanishes if one of them is equal to zero.

The quantitative evaluation of the obtained expressions depends on many factors. The selection of the potential involved in the spin-orbital interaction plays a major role in the calculation of the effective cross section and also the polarization vector [9]. The angular distribution of scattering of particles is equally well described by different sets of optical potentials. But an unambiguous determination of the parameters of the potentials of an optical model only from an analysis of the angular distributions of elastic scattering is in principle impossible, since the elastic scattering of complex particles is determined not only by potential scattering.

3. Conclusion

The study of nuclear- particle interactions is often used as a powerful tool for obtain information about local symmetry. The symmetry structure of the nuclear system is in general very complex. In addition, the process by which radiation occurs also complicates the problem.

In two-nucleon reactions, unlike simpler one-nucleon reactions, we cannot obtain the spectral amplitude from an experiment. For the low-lying collective states, these spectral amplitudes, generally speaking, can be predicted only in some limiting cases, when the corresponding nuclei are either spherical or deformed, whereas in the transitive region, no satisfactory formalism has been developed for the nuclei.

When unpolarized particles are scattered by nuclei, the elastically scattered particles are polarized. Since the scattering amplitude contains a term that depends on the spin orientation and a term that does not depend on the spin orientation, the polarization is due to the interference between these two parts of the scattering. Large values of the degree of polarization (the absolute value of the polarization vector is called the degree of polarization) are possible when the magnitude of the interference is comparable to the magnitude of the scattering cross section.

Existing methods and approximations, such as the distorted wave method, the coupled channel method, and the momentum approximation are insufficient to explain the polarization data. The inclusion of higher order terms and the connection with statistical processes requires more work. In connection with pre-equilibrium effects and the emission of fast light associations, the transition to the consideration of three-particle channels is inevitable.

The possibility of describing the interaction between nucleons with the help of a potential is limited from the side of high energies. Such a limitation in energy means that an unambiguous determination of the potential at small distances is generally impossible, and this strongly affects the values of the polarization. However, since the maximum allowable values of nucleon momenta in nuclei correspond to energies below 450 MeV, one can hope that relativistic effects can be neglected when considering the structure of nuclei.

References

1. J.L. Wood, J.Phys.: Do we understand excited 0+ states in nuclei? Conf. Ser., 2012, 403, 012011 DOI: 10.1088/1742-6596/403/1/012011.
2. W.D. Kulp et al., Search for intrinsic collective excitations in ^{152}Sm. Phys. Rev. C, 2008, 77, 061301(R).
3. D.A. Meyer et al., Phys. Rev. C, 2006,74, 044309.
4. K. Heyde and J.L.Wood, Shape coexistence in atomic nuclei. Rev. Mod. Phys., 2011, 83, 1467.
5. C. Bernards, R.F. Casten, V. Werner Investigation of 0+ states in mercury isotopes after two-neutron transfer. Proceedings, 25th International Nuclear Physics Conference (INPC 2013): DOI: 10.1051/epjconf/20146602008
6. S.R. Lesher et al., Low energy 0+ excitations in 158Gd.AIP Conf. Proc., 610, 798 (2002).
7. I. G. Afandıyeva. Electric Quadrupole Transitions in Processes With Stripping Bineutrons Clusters. Cumhuriyet Science Journal, (2017), 38 (3) , 444-449 . DOI: 10.17776/csj.340478
8. R.A Ahmedov. Polarization effects in hadron-nuclei scattering at high energy. Cumhuriyet University Faculty of Science Journal (CSJ), 2016, Vol. 37, p. S56-S58 ISSN: 1300-1949.
9. K. Blum and B. Lohmann. Entanglement and Bell Correlation in Electron-Exchange Collisions.Phys. Rev. Lett. 2016. 116 033201. DOI:10.1103/PhysRevLett.116.033201.

ПОЛЯРИЗАЦИОННЫЕ ЭФФЕКТЫ В КВАДРУПОЛЬНОМ ИЗЛУЧЕНИИ

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Резюме: Исследованы поляризационные эффекты в процессах квадрупольного излучения. Условие унитарности использовалось для определения амплитуды рассеяния. Эффект поляризации существенно зависит от относительной величины членов амплитуды. Поляризация исчезает, если один из этих членов равен нулю.

Ключевые слова: квадрупольное излучение, эффективное сечение, поляризация, спин-орбитальное взаимодействия

KVADRUPOL ŞÜALANMASINDA POLYARLAŞMA EFFEKTLƏRİ

İ.Q. Əfəndiyeva, R.A. Əhmədov

Xülasə: Kvadrupol şüalanması proseslərində səpilən zərrəciklərin polyarlaşma effektləri tədqiq edilmişdir. Səpilmə amplitudunu təyin etmək üçün unitarlıq şərtindən istifadə edilmişdir. Polyarlaşma effekti mahiyyətcə amplitudun hədlərinin nisbi qiymətindən asılıdır. Bu hədlərdən biri sıfır olarsa polyarlaşma itir.

Açar sözlər: kvadrupol şüalanması, effektiv kəsik, polyarlaşma, spin-orbital qarşılıqlı təsir