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MULTIPOLE RADIATIONS IN DIRECT NUCLEAR REACTIONS

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Abstract: Based on the theory of distorted plane waves, multipole transitions in direct reactions are investigated. A technique has been developed for taking into account the effects of the Coulomb interaction in the analysis of direct nuclear reactions. It is shown that taking into account the Coulomb effects significantly affects the differential cross section for the reactions, the values of quadrupole radiation and improves the agreement between theory and experiment. The results are given in the work related to the analysis of the matrix element of the electric quadrupole transitions.

Keywords: radiation, direct nuclear reaction, Coulomb interaction, electric quadrupole transitions

1. Introduction

Experimental and theoretical study of direct reactions is provided by a powerful method of reaction spectroscopy. If we had a good theory about the mechanism reactions, we would obtain certain information about short-range spatial correlations between nucleons in nuclei. The problem of direct nuclear scattering, the simplest one in the problem of nucleon - nuclear interaction, has been studied for a long time and very effective calculation schemes have been developed for its solution. However, the main features of the problem, primarily the physical picture of the phenomena, remain insufficiently studied in comparison, for example, with the usual potential problem of the scattering of two particles. The difficulties in studying them are mainly due to the extraordinary complexity of the general analytical analysis of solutions in the many-particle problem.

If the need to introduce a finite interaction radius for calculating exchange processes is more or less obvious, then it would seem that for calculating direct processes in reactions with light particles, one can restrict oneself to a zero interaction radius. However, this is not the case, since even in single-nucleon transfer reactions with light particles on light nuclei the recoil of the entire system is not small. If we consider the transfer reactions of several nucleons (for example, (p, t), (t, p), etc.), then the effects associated with recoil become even more important. From this point of view, the use of a zero radius of interaction is most justified in the reactions (d,p), (d,n), since here the recoil is the least significant.

In this paper, we obtained the expression for matrix element for electric quadrupole radiation, the energies of 0^+ states, the probabilities of the E(2) transition and effective cross sections of (p,t) and (t,p) reactions for the nuclei Sm^{154} and Sm^{156} are calculated. The main formulas for cross section (*p*,*t*) and (*t*,*p*) reactions are given in [1].

2. Electric quadrupole matrix element

These motions enhance the electric quadrupole matrix elements and give the events a unique signature, which can be used for identification. The study shows a strong population in (p,t) reactions of the 0^+ excited states in the rare-earth regions can be associated with the density distribution of single - particle levels and the alignment of the corresponding quadrupole moments in the vicinity of the Fermi surface of these nuclei. So the corresponding transfer matrix element is diagonal in the spin states of the nucleus. In the direct transfer reactions, the low-lying 0^+ levels will be more strongly excited in nuclei in which the single-particle quadrupole transitions near the Fermi surface have the same sign [2].

The effect of direct interaction becomes significant in the energy range of the relative motion of the nucleon and nucleus, which is in the range of 10-40 MeV. At energies exceeding 50 MeV, the "transparency" of the nucleus increases and interaction becomes possible with the transfer of energy to one nucleon sufficient to remove it from the nucleus, and this nucleon can be not only a "surface", weakly bound nucleon, but also a "deep" one.

It should be noted that the complexity in the structure of excited 0^+ states springs up a variety of approaches ranging from microscopic studies of anharmonic effects, consideration of quadrupole and pairing vibrational modes in direct nuclear reactions [3]. On the other hand, the description of the energies and properties of the states are important tests in the evaluation of the applicability of the different models, like the shell model, cluster- vibrational model, quasi-particle - phonon model.

The problem of calculating the energy and other characteristics of the system of identical particles encounters difficulties in constructing the wave function. In theory, the reaction is considered as a quantum mechanical process due to the fact that different components of the wave function of the incident hadron have different probability of interaction with the target [4]. As a result, the wave function is distorted. The Born approximation with distorted waves has long been used to analyze rearrangement reactions in the theory of direct nuclear reactions.

We will discuss some aspects of direct collisions without physical overlap, i.e. collisions with impact parameters b larger than the sum of the nuclear radii, R, i.e. $b > 2R$. Particles can be produced in these collisions through an interaction of the fields of the nuclei. If the momentum transfers from the nuclei are small enough ($q<kc/R$), the fields of nucleus and incident particle interacted coherently to all nucleons.

To describe incident particles, a distorted $\Psi^+(r)$ wave should be used, at infinity a "plane plus divergent wave":

$\Psi^+(r) = e^{ik_1 r}$ + divergent wave,

to describe outgoing particles - a distorted $\Psi^-(r)$ wave, at infinity a "plane plus a converging wave":

$\Psi^-(r) = e^{ik_2 r}$ + converging wave.

Since both incident and target particles have an electric charge, the scattering is caused by both nuclear forces and Coulomb forces. Coulomb forces play a key role in the low values of the relative movements of incident particle, and nuclear forces play a key role in their large values.

The amplitude in the theory of distorted waves has the form:

$$F(k_1, k_2) = -\frac{\mu}{2\pi\hbar^2} \int \Psi_{k_2}^{(-)^*}(r)\left[V_N + V_C\right]\Psi_{k_1}^{+}(r) d^3 r \, , \qquad (1)$$

where k_1 is the momentum of the incident particle, k_2 is the momentum of the emitted particle.

The structural form factor included in the cross section of any process is directly expressed in terms of the square of the modulus of the reduced matrix element from the corresponding multipole moment of the nucleus. Taking into account the charge degrees of freedom of nucleons, the nucleus can be described by multipoles of sixteen types of given multipoleness. Each transition of certain multipolarity corresponds to its strictly defined selection rules. The use of the multipole formalism facilitates the corresponding calculations and makes it possible to clarify the physical nature of various transitions in nuclei.

In the distorted-wave approximation, the differential cross section for inelastic scattering is calculated by the formula

$$\frac{d\sigma}{d\Omega} = \frac{\mu^2}{4\pi^2\hbar^4} \int \Psi_{k_2}^{(-)^*}(r)\Psi_f^*(\xi)\left[V_N + V_C\right]\Psi_i(\xi)\Psi_{k_1}^{+}(r)d\xi d^3r \, . \tag{2}$$

The main feature of the integrals appearing in expressions (2) is that the wave function of the relative movement and distorted waves also depend on the relative different combinations of variables.

At energies of 10 MeV, specific nuclear forces affect only the *S* (*l*=0) wave and cause a phase shift. The remaining partial waves will only have a shift due to the Coulomb interaction, because at such low energies electric charge particles cannot approach each other until the nuclear force field due to the Coulomb sharpening force.

The two-nucleon transfer reactions also, have taken on great importance, because they are to be very sensitive to nuclear spectroscopy. The task of the (p,t) or (t,p) scattering is the simplest problem in the nucleon - nucleus interaction or interaction of a particle with a complex structure, which has been studied for a long time and its solution, developed highly effective calculation schemes.

In particular, we consider the structure of the angular momentum transfer that occurs during the two-nucleon transfer reaction *A*(p,t)*B*. First, we make the widely used assumption that only the "direct" term contributes (meaning that we do not take into account the exchange of nucleons between two nuclei) and that the target nucleons are not excited. Then, we restrict ourselves to most of the usual cases of nuclear reactions, in which the internal states of the incoming p, outgoing t, and any intermediate particles (c=d=p+n) that are formed during the successive transfer are assumed to be completely symmetrical S-states, so that the corresponding successive transfer interactions are diagonal in the spin states of the nuclei. We, naturally, believe in this reasoning that the distorted radial wave functions of two neutrons captured by a proton are close to each other, in particular in the peripheral region of the nucleus, where the "crosslinking" of the wave functions of these nucleons occurs in the target nucleus *A* and in the nucleus *B*.

The assumption formulated above is a natural generalization of what we know about the interaction of two nucleons in the final state in reactions of the (p,t) type. This interaction is significant only when the wave vectors of the two nucleons are close to each other.

Briefly, the first order transition occurs directly from the initial to the final state by the transfer of a nucleon pair, without changing the internal nucleons. It is assumed that the transferred neutrons are in the bound *S*-state.

Let us consider the calculation of the matrix element for the multipole transition $J_i \to J_f$. Matrix element has following expression:

$$M = \frac{\left\langle \Psi_{J_f 0}^* \middle| \hat{Q} \middle| \Psi_{J_i 0} \right\rangle}{\sqrt{N_i N_j}}, \tag{3}$$

where $\Psi_{J_i 0}$ and $\Psi_{J_f 0}$ - destroying initial and final state functions, N_i and N_j the normalization integrals.

The quadrupole transition operator in the Jacobi variables can be written as follows:

$$\hat{Q} = 2R^2 Y_{20}(r) + \sum_{l=1}^{A} \left\{ r_l^2 Y_{20}(r_l) + \frac{1}{2} u_l^2 Y_{20}(u_l) + + \frac{1}{2} v_l^2 Y_{20}(v_l) \right\}, \tag{4}$$

where the Jacobi variables describe the motion of two pick up or captured particles:

$$u_l = r_{l1} - r_{l2};\ v_l = r_{l1} + r_{l2}. \tag{5}$$

The term containing the quadrupole moment of the center of mass of the nucleus is discarded, since it does not correspond to any real physical transition of the nucleus from one state to another, but leads to excitation of the center of mass of the nucleus.

Substituting (4) and (5) into (3) for the matrix element, after using the diffraction theory, we obtain expression for matrix element for quadrupole radiation:

$$M_{i\to f}^{J=2} = \delta_{m_t m_p} \frac{\delta_{m_t m_p}}{\sqrt{N_{00} N_{20}}} \sum_L 4\pi \sqrt{(2L+1)} i^{l+2} \int \Psi_{20}^{*J=2}(\xi, \vec{R}) \Psi_{00}(\xi) j_l(k_p \vec{R}') j_L(k_t \vec{R}) d\xi d\vec{r} d\vec{R}' d\vec{R}. \tag{6}$$

Experimental data [5-7] and calculation results are summarized in Table 1.

As can be seen from Table 1, taking into account the Coulomb interaction leads to an underestimation of the value of the effective reaction cross section for the first excited state. As can be seen from the table, the calculated energies turned out to be higher than the experimental ones, which suggest the noncollective nature of these states. For the first excited state, the values $B(E2)$ and effective cross sections of the reactions agree satisfactorily with experiment. As the energy increases, the values of $B(E2)$ and effective cross sections are small. For the considered nuclei, the effective cross sections in the reactions (p,t) and (t,p) have rather close values.

An increase in the cross section for reactions of two-nucleon transfers to the ground states of even-even nuclei, as compared to transitions to a pure shell configuration, can be associated with the correlation of pairs of nucleons in the ground state of the final nucleus.

When compared with experiment, an important role is played by the choice of deformation parameters, which can significantly affect the excitation of 0^+ states in the reactions (p, t) and (t, p). In the nuclei of the transition region, a strong population of the excited 0^+ states in the reactions (p,t) and (t,p) occurs when the deformation of the excited state of the daughter nucleus coincides with the deformation of the ground state of the parent nucleus.

Table 1

Theoretical and experimental ratios of the $\frac{B(E2; I+2\to 2)}{B(E2; 2\to 0)}$ values for nuclei ^{154}Sm and ^{156}Sm. The experimental dates give from [5-7].

	Nucleus	Sm^{154}				Sm^{156}			
Theory	ω (MeV)	1,08	1,43	2,52		1,06	2,16	2,24	
	$B(E2)_{s.p.u.}$	4,45	0,09	0,01		7,51	0,02	0,22	
	$\sigma(p,t)/\sigma_0$	1,43	0,08	<0,01		1,31	0,02	<0,01	
	$\sigma(t,p)/\sigma_0$	1,16	0,42	0,01		1,46	0,22	<0,01	
Experiment	ω (MeV)	1,10	1,22	-	-	1,07	-	-	-
	$B(E2)_{s.p.u.}$	1,2	<0,01	-	-	-	-	-	-
	$\sigma(p,t)/\sigma_0$			-	-	-	-	-	-
	$\sigma(t,p)/\sigma_0$	0,10	0,33	-	-	0,07	-	-	-

3. Conclusion

There are different approaches to explaining multipole radiation, each of which correctly treats some, but not all aspects of this radiation. Among these aspects: the use of the destroyed wave functions of particle and the corresponding exact interaction; accurate interpretation of the limited domain of interactions in the amplitude. In addition, all the results are very sensitive to potentials and the wave functions of the nucleus used for the transition matrix.

References

1. S.G. Abdulvahabova, N.Sh. Barkhalova and. T.O. Bayramova, Studying the E2 Transitions in the Representation of SU(5) Subgroup. Proceedings Star-Net. Modern Trends in Physics, 2019, 253-255.
2. , S.K. Abdulvagabova, S.P. Ivanova and N.I. Pyatov, Excitation of 0^+ states in two- particle transfer reactions. Phys.Lett. B, 1972, 38, 215-217.
3. J.L. Wood, Do we understand excited 0^+ states in nuclei? Conf. Ser., 2012, 403, 012011 DOI: 10.1088/1742-6596/403/1/012011.
4. N.P. Zotov and V.A. Tsarev, Diffraction dissociation. Phys. Uspekhi, 1988, 31 (2), p. 119-139.
5. W.D. Kulp, et al. Search for intrinsic collective excitations in ^{152}Sm. Phys. Rev. C, 2008, 77, 061301(R).
6. D.A. Meyer, et al. Phys. Rev. C, 2006, 74, 044309.
7. K. Heyde and J.L. Wood, Shape coexistence in atomic nuclei. Rev. Mod. Phys., 2011, 83, 1467.

МУЛЬТИПОЛЬНЫЕ ИЗЛУЧЕНИЯ В ПРЯМЫХ ЯДЕРНЫХ РЕАКЦИЯХ

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Резюме: На основе теории искаженных плоских волн исследуются мультипольные переходы в прямых реакциях. Разработана методика учета эффектов кулоновского взаимодействия при анализе прямых ядерных реакций. Показано, что учет кулоновских эффектов существенно влияет на дифференциальное сечение реакций и на значения квадрупольных излучений и улучшает согласие теории с экспериментом. Результаты приведены в работе, связанной с анализом матричного элемента электрических квадрупольных переходов.

Ключевые слова: излучение, прямая ядерная реакция, кулоновское взаимодействие, электрические четверные переходы.

BİRBAŞA NÜVƏ REAKSİYALARINDA MULTİPOL ŞÜALANMALARI

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Xülasə: Təhrif olunmuş müstəvi dalğalar nəzəriyyəsinə əsasən birbaşa reaksiyalarda multipol keçidləri tədqiq olunur. Birbaşa nüvə reaksiyalarının təhlilində Kulon qarşılıqlı təsirinin nəzərə alınması üçün üsul işlənib hazırlanmışdır. Göstərilmişdir ki, Kulon effektlərinin nəzərə alınması reaksiyaların diferensial kəsiyinə, kvadrupol şüalanmasına əhəmiyyətli dərəcədə təsir edir və nəzəriyyə ilə təcrübə arasında uyğunluğu yaxşılaşdırır. Işdə nəticələr elektrik keçidlərinin matris elementinin təhlili ilə bağlı təqdim olunur.

Açar sözlər: radiasiya, birbaşa nüvə reaksiyası, Kulon qarşılıqlı təsiri, elektrik kvadrupol keçidləri.